D II Physics Hons (Paper - IV)

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~~Electrostatics:~~

Poisson's and Laplace's Equations:

We know that the electric field may be expressed as negative gradient of potential. That is,

$$\vec{E} = -\,\text{grad}\,V = -\vec{\nabla}V.$$

From Gauss we know that

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}, \quad \text{--- (i)}$$

Where ρ is the volume density of charge. Making the substitution $\vec{E} = -\vec{\nabla}V$ in eqn (i), we get

$$\vec{\nabla}\cdot(\vec{\nabla}V) = -\frac{\rho}{\epsilon_0}. \quad \text{---- (ii)}$$

Now
$$\vec{\nabla}\cdot\vec{\nabla} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\cdot\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$= \nabla^2$$

Where ∇^2 is known as Laplacian operator. The eqn. (ii) becomes

$$\boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}}$$

or, div grad $V = -\frac{\rho}{\epsilon_0}$

This equation is known as Poisson's equation. It expresses a relation between the potential V and the charge density ρ at any point in an electric field in space. Any static electric field must satisfy this relationship.

The Poisson's equation in cartesian coordinates is

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

In a region where there are no free charges ($\rho = 0$), the Poisson's equation reduces to

$$\boxed{\nabla^2 V = 0}$$

or, div grad $V = 0$.

This is called Laplace's equation. In Cartesian coordinates, we may write it as

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$